Transcomputation - Answers 6

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Note

In this Exercise polar-transcomplex numbers are written in parentheses as trans-tuples of the form (r, θ), where r and θ are transreal numbers, and Cartesian transcomplex-numbers are written in square brackets as trans-tuples of the form $[x, y]$, where x and y are transreal numbers.

1 Transcomplex sums

1.1 Here $\cos(\theta)$ and $\sin(\theta)$ are total functions. Because they are total, every value of θ maps to some value(s) and because they are functions, they map to one value. Multiplication is an operator, so every product of r with the unique value of $\cos(\theta)$ and $\sin(\theta)$ is unique.

1.2 $[\infty, 1] \neq [\infty, 2]$ but both tuples correspond to $(\infty, 0)$. To see this, observe that the polar form of $[\infty, 1]$ has an angle, θ, given by $\tan\theta = 1/\infty = 0$, whence $\theta = \tan^{-1} 0 = 0$ and, similarly, $[\infty, 2]$ has $\tan\theta = 2/\infty = 0$, whence $\theta = \tan^{-1} 0 = 0$.

1.3 Here $a = (2, 0)$ corresponds to $a' = [2, 0]$ and $b = (2, \pi/4)$ corresponds to $b' = [2\cos(\pi/4), 2\sin(\pi/4)] = [2/\sqrt{2}, 2/\sqrt{2}] = [\sqrt{2}, \sqrt{2}]$.

1.4 Now $c' = a' + b' = [2, 0] + [\sqrt{2}, \sqrt{2}] = [2 + \sqrt{2}, 0 + \sqrt{2}] = [2 + \sqrt{2}, \sqrt{2}]$.

1.5 Now $c = (r, \theta)$ has $r = \sqrt{\{2 + \sqrt{2}\}^2 + \sqrt{2}^2} = \sqrt{\{2^2 + 4\sqrt{2} + \sqrt{2}^2\} + \sqrt{2}^2}$ $= \sqrt{4 + 4\sqrt{2} + 2 + 2} = \sqrt{8 + 4\sqrt{2}} \simeq 3.7$. And it has $\tan\theta = \sqrt{2}/(2+\sqrt{2})$, whence $\theta = \tan^{-1}(\sqrt{2}/(2 + \sqrt{2})) = \pi/8$.

1.6 We could use the transcomplex cylinder to compute the result, but it is quicker to observe that the sum is the bisector $(\infty, 0.5) + (\infty, 0.6) = (\infty, 0.55)$.

1.7 The sum is the bisector $(\infty, 0.5) + (\infty, -0.5) = (\infty, 0)$.

1.8 We could use the transcomplex cylinder to compute the result, but it is quicker to use the properties of nullity. Thus $(\Phi, 3) + (\infty, 6) = (\Phi, 0) + (\infty, 6) = (\Phi, 0)$.

1.9 Similarly $(2, \infty) + (3, 4) = (2, \Phi) + (3, 4) = (\Phi, 0) + (3, 4) = (\Phi, 0)$.

2 Transcomplex division

2.1 We are required to prove that the division formula, $(r_1, \theta_1) \div (r_2, \theta_2) = (r_1/r_2, \theta_1 - \theta_2)$, calculates infinity correctly if and only if the angle of zero is zero. We begin by noting that infinity is given in its most general transreal form as $\infty = k/0$, where k is a strictly positive real number and 0 is real zero. We observe that the polar form of transreal ∞ is $(\infty, 0)$, where the elements of the tuple are transreal numbers. And the polar form of transreal k is $(k, 0)$, where the elements of the tuple are real numbers. Let transreal 0 have polar form $(0, \theta)$, where the elements of the tuple are transreal numbers. Corresponding to transreal $\infty = k/0$ we have polar $(\infty, 0) = (k, 0) \div (0, \theta) = (k/0, 0 - \theta) = (\infty, -\theta)$. Hence $-\theta = 0$, as required. This proves that in transcomplex arithmetic, the angle of zero is zero: $0 = (0,\ 0)$, where the left hand side is transreal zero and the right hand side is transcomplex zero. By contrast, in complex arithmetic it is only a convention that the angle of zero is zero, but in transcomplex arithmetic it is a theorem. Henceforth we always reduce the angle of zero to zero before operating on it.

2.2 Corresponding to transreal $\Phi = 0/0$ we have polar $(\Phi, \theta) = (0, 0) \div (0, 0) = (0/0, 0 - 0) = (\Phi, 0)$. Hence $\theta = 0$, as required. Other angles of nullity can occur, but this computation justifies the conventional zero angle of nullity. Henceforth we always reduce nullity to conventional or standard form before operating on it.

Take care! If a user or some calculation path gives us $x = (0, \theta)$, for any real θ, then the proof in (2.2) requires us to write $x = (0, 0) = 0$. But if θ is strictly transreal then $x = \Phi$ because of the equivalence $(\Phi, \theta) = (r, \Phi) = (r, -\infty) = (r, \infty)$, for all transreal θ and r, including $r = 0$. In the case $x = \Phi$ we write $x = (\Phi, 0)$, but the tuple $(\Phi, 0)$ is just the conventional one of many equivalent tuples.